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FROM EFFICIENCY MEASUREMENT TO EFFICIENCY IMPROVEMENT: THE CHOICE OF A RELEVANT BENCHMARK

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Abstract

This article deals with efficiency improvement and how to identify appropriate benchmarks for inefficient firms to imitate. We argue that the most relevant benchmark is the most similar efficient firm. Having interpreted similarity in terms of input endowments, the problem reduces to find the closest reference firm on the efficient subset of the isoquant. To such an end, we introduce the concept of input-specific contractions. This concept allows to find the shortest path to the efficient subset. This information can be used to advise inefficient firms about which efficient firm to visit in order to detect its mistakes and to learn better managerial practices.

Keywords. Data Envelopment Analysis, Benchmarking, Shortest path, Efficiency, Management

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1. Introduction

Technical inefficiency reflects the failure of some firms to obtain the maximum feasible output given the amount of inputs used. Its measurement is crucial to quantify the importance of poor performances in a productive activity. Unfortunately, measurement is not enough. In order to improve technical efficiency (TE), firms should be able to identify the sources of misperformances and the alternatives available to make better use of their resources. Therefore, the question to be answered is "how can a firm become efficient in practice?". The answer to this question depends on the sources of inefficiency.

Some studies consider technical inefficiency as the result of a lack of motivation or effort, as suggested by Leibenstein (1966). Thus, the question of efficiency improvement is assessed within the framework of principal-agent contractual theory. In this line, Bogetoft (1994) suggests that efficiency improvements may be achieved introducing an appropriate incentive scheme to induce the desired (efficient) effort level from the agent. A different approach considers technical inefficiency as the result of a lack of knowledge or managerial ability (Farrell, 1957). Under this view, efficiency improvements may be achieved through learning processes, as is the case of management programs. Thus, the main difference between the two approaches is the assumption made about the motivation of the productive agents.

This paper is grounded on the knowledge-based view of efficiency, which focuses on firms that are inefficient, but have the motivation needed to become efficient. The objective is to provide an insight in efficiency improvement through learning processes. We argue that, if knowledge is the main driver of inefficiency, firms should try to learn from those that are efficient. Then, we develop a method to identify the appropriate benchmark each firm should try to learn from.

The paper is organized as follows. Section 2 explores how a firm can improve efficiency in practice. Section 3 reviews the current efficiency measures. In Section 4, the concept of input-specific contraction is introduced. An empirical non-parametric model is presented in Section 5. Finally, concluding remarks are presented.

2. Efficiency Improvement in Practice

Some common questions posed by inefficient farmers enrolled in farm management programs are How can I become efficient? or What am I doing wrong?. However, current measures of TE do not provide information of this kind. The implicit assumption in TE studies is that inefficient firms should behave as those on the best practice frontier. But behavior has two components: how much the firms are doing and how they do it. Efficiency measures only inform in terms of “how much”. The firms on the frontier determine the input reduction an inefficient firm could achieve. However, if this inefficient firm reduces its input use but continues acting in the same manner as before the reduction, we would observe a firm that produces less output and is equally inefficient.

We can illustrate this problem with an example. Assume two farmers with the same type of farm, which apply the same amount of fertilizer to one hectare of land. Farmer A applies the fertilizer at the appropriate period of the year while farmer B does not. Other things being equal, farmer A will produce a higher amount of output than farmer B and, therefore, our current efficiency measures will label farmer B as inefficient. After this finding, one might be prompted to conclude that farmer B could (should) reduce its input use and still produce the same amount of output. However, after downsizing, farmer B will still be applying fertilizer with bad timing and therefore he will still be inefficient.

Obviously, the solution for the inefficient firm is to find out what it is doing wrong and then correct its mistakes. The question is how to do this in practice. A reasonable strategy would be that, after the firm is informed that it is inefficient, its manager visits some of the efficient firms to observe how they do things. This benchmarking procedure is common in farm management programs. A non-trivial question here is how to choose which of the efficient firms it should visit. Even though this is done in real practice in several ways, the purpose of this paper is to provide a method to identify this subset of most relevant firms to visit in a more objective way.

It seems natural to think that an inefficient firm will prefer to visit the efficient firm that is most similar to it, rather than an efficient but very different firm. Research on interorganizational learning supports this idea. As Lane and Lubatkin (1998) put it "(...) the ability of a firm to learn from another firm (...) depends upon the similarity between the student and the teacher firms". The most similar the efficient firm, the easier it will be for the inefficient firm to detect its own mistakes and, therefore, to correct them. But to make this idea operative, we must find a definition of firm similarity.

Most empirical studies of TE use radial measures to quantify efficiency. Thus, it may be argued that the most similar firm is the radial projection of the inefficient firm on the isoquant. Radiality seems to be a reasonable proxy for similarity, because all firms on the same ray share the same combination of inputs[1]. However, it is easy to imagine a situation in which two firms sharing the same input proportions may be quite different. Furthermore, it has been noticed that radial measures impose a direction for improvement that does not take into account the information on input substitution possibilities that is available through the empirically constructed isoquant (Bogetoft and Hougaard, 1999).

A better criterion for practical purposes may be proximity, which can be measured in terms of inputs. This practice would be more in the line of traditional cluster analysis, a technique that is typically used to find patterns of similarity among observations. While aware that this is not a scientific criterion, an inefficient firm could be more interested in visiting a firm that uses more or less the same quantities of inputs (it is in the same input cluster) than in visiting a firm that is using the same proportion of inputs but at a different scale. The literature on farm management provides some examples of the use of this criterion of similarity. For example, Lund and ∅rum (1997) have developed a computerized efficiency analysis system, for management advisory purposes, that compares each firm with a reference group composed of the most similar firms in terms of absolute quantities of certain inputs. Dervaux, Kerstens, and Vanden Eeckaut (1998) suggest that a modified version of Färe's (1975) asymmetric efficiency measure, defined as the smallest input contraction needed to reach the isoquant, can be interpreted as "the minimal effort required to join the boundary of a technology". Similarly, Frei and Harker (1999) have proposed a least norm distance measure to the production frontier, allowing innefficient firms to benchmark against those efficient firms that most closely resemble them. Although useful, a problem with this least norm distance is that it "is not invariant with respect to the scale of the units used for the inputs and/or outputs" (1999: p. 292).

In this article, we go a step further, trying to provide an operative way to find the closest reference firm in the efficient subset of the isoquant using relative contractions of inputs and, thus, avoiding the units of measurement problem mentioned above. The comparison group on the efficient subset will be composed of efficient firms that share the largest number of similarities in the input endowments and, therefore, are easier to imitate by the inefficient firm. The idea is that the inefficient firm may learn more from visiting these firms than visiting any other efficient firms. To make this idea operative, we introduce the concept of *input-specific contraction* as a modified version of the *single-factor efficiency* measure, introduced by Kopp (1981). The input-specific contraction measure computes the sum of input contractions required to reach the efficient subset of the production frontier when the contraction to the isoquant is measured along a single input.

3. Measures of Technical Efficiency

The technology can be characterized by the input requirement set, $\mathbf{u} \rightarrow L(\mathbf{u})$, where $L{:}\Re_+^S \rightarrow \Re_+^N$ is a mapping from the output vector $\mathbf{u} \in \Re_+^S$ into the set of input vectors $\mathbf{x} \in \Re_+^N$ that allow to produce **u**. Throughout the paper, we will assume that $L(\mathbf{u})$ satisfies the properties of convexity, free disposability of inputs and outputs and variable returns to scale.

Koopmans (1951) defines an input-output vector (IOV) as technically efficient if, and only if, increasing any output or decreasing any input is possible only by decreasing some other output or increasing some other input. Based on the previous definition, technical efficiency measures evaluate the performance of a given IOV by comparison to the IOVs on the boundary of $L(\mathbf{u})$.

Two boundary sets are relevant for the measurement of TE. The input isoquant is defined as[2]:

$$Isoq\ L(\mathbf{u}) = \left\{\mathbf{x} \in \Re_+^{\mathbf{N}} : \mathbf{x} \in L(\mathbf{u}) \wedge \lambda\mathbf{x} \notin L(\mathbf{u}),\, \lambda \in [0,1)\right\} \tag{1}$$

and the efficient subset of the isoquant is defined as[3]:

$$Eff\ \ L(\mathbf{u}) = \left\{\mathbf{x} \in \Re_+^N : \mathbf{x} \in L(\mathbf{u}) \wedge \hat{\mathbf{x}} \le \mathbf{x},\, \hat{\mathbf{x}} \notin L(\mathbf{u})\right\} \tag{2}$$

Radial measures of TE carry the comparison along a ray from the origin and are attractive because they maintain the input mix of the IOV onto its projection on the boundary of the input requirement set. Therefore, they have a direct interpretation in terms of proportional cost reduction. The Farrell (1957) radial index focuses on the maximum equiproportionate reduction in all the inputs that can be achieved holding constant the output vector, and is defined by:

$$F(\mathbf{x},\mathbf{u}) = \min_{\theta}\left\{\ \theta : \theta\ \ \mathbf{x} \in L(\mathbf{u}), \theta \in \Re_+\right\} \tag{3}$$

However, $F(\cdot)$ is not always consistent with Koopmans' definition, because the comparison is done with respect to the isoquant and not with respect to the efficient subset. An IOV on the isoquant is considered efficient although it may remain slacks in some inputs.

Non-radial measures of TE avoid this problem by restricting the comparison to the efficient subset. The Russell measure introduced by Färe and Lovell (1978) satisfies this property and is defined as:

$$R(\mathbf{x},\mathbf{u}) = \min_{\boldsymbol{\theta}}\left\{\frac{\sum_{n=1}^{N}\theta_n}{N} : \left(\theta_1 x_1, \cdots, \theta_N x_N\right) \in L(\mathbf{u}),\quad \theta_n \in [0,1]\ \forall n\right\} \tag{4}$$

This measure shrinks the input vector not along a ray, but in coordinate directions until a point in the efficient subset of the isoquant is reached. The difference between the two measures is illustrated in Figure 1.

<<<<<<<<<<<FIGURE 1 ABOUT HERE>>>>>>>>>>>>>

For unit E, the Farrell measure of efficiency is given by the ratio OH/OE. The comparison point on the isoquant (H) lies on the same ray of unit E, thus maintaining the proportions in the input mix. To compute the Russell measure, the comparison point must lie on the efficient subset (ABC). By construction, the reference point is A and the measure takes the value 1/2(IG/IE + DA/DG), which represents the maximum average contraction that is feasible.

The indexes discussed above are called by Kopp (1981) *multiple-factor efficiency* measures, as they encompass the efficiency of total factor use[4]. Kopp introduces the notion of *single-factor efficiency* measures as an attempt to understand the individual contribution of each input to inefficiency. The single-factor efficiency measure of input k is given by:

$$K_k(\mathbf{x},\mathbf{u}) = \min_{\theta_k}\left\{\theta_k : (x_1,\cdots,\theta_k x_k,\cdots,x_N) \in L(\mathbf{u}) \,,\quad \theta_k \in \Re_+\right\} \tag{5}$$

Expression (5) gives the contraction in input k needed to reach the isoquant and can be interpreted as the lower bound in the efficiency with which that input is used. In Figure 1 the Kopp measure takes the value FB/FE for input X_2 and IG/IE for input X_1. It is worth noting that the comparison set is the isoquant and not necessarily the efficient subset.

Single-factor efficiency measures are interesting because they focus on the contraction of the IOV to *Isoq* $L(\mathbf{u})$ along a unique coordinate direction. The smallest of Kopp indexes reflects the input that needs the largest contraction to reach the isoquant. This smallest index is what Färe (1975) names *input efficiency function*. Symmetrically, the largest of Kopp indexes reflects the input that needs the smallest contraction and, therefore, requires the minimal effort to reach the isoquant (Dervaux, Kerstens, and Vanden Eeckaut, 1998).

Bogetoft and Hougaard (1999) have proposed a measure that stresses the importance of selecting the benchmark on the isoquant. The so called "potential improvements index" selects the vector constructed after substracting from each input its largest possible reduction (the Kopp saving) as the one defining the reference direction to the efficient subset. The advantage of this measure is that the information about the shape of the isoquant is considered in the selection of the reference efficient point, as it is based on potential improvements in all inputs. However, the reference point on the production frontier is not the closest one. Frei and Harker (1999) have developed a procedure to compare against the closest efficient benchmark, which is based in computing the smallest norm to the production frontier. However, their measure is sensitive to the units of measurement used. In the next section we provide a way to overcome this problem by using input specific contractions instead of absolute input reductions.

4. Input Specific Contraction to the Efficient Subset

In Section 2, we reviewed various measures of TE that share the common feature of quantifying the relative proportions by which a firm could reduce its inputs. The magnitude of such reductions may be an important factor in designing the best strategy to achieve efficiency. In other words, the task of reducing inputs while maintaining output levels is not trivial and implies considerable efforts (if not, we would not observe inefficient firms). The easiest manner to achieve efficiency may consist in visiting similar firms that are efficient. Therefore, a procedure to identify the closest efficient firm can help the design of a plan towards efficiency achievement.

Definition 1. We define the smallest contraction to the efficient subset as the minimum contraction in the inputs required to reach the efficient subset:

$$C(\mathbf{x},\mathbf{u}) = \min_{\theta}\left\{\sum_{n=1}^{N}(1-\theta_n) : (\theta_1 x_1, \cdots, \theta_N x_N) \in Eff\, L(\mathbf{u}) \quad , \quad \theta_n \leq 1 \quad \forall n\right\} \tag{6}$$

Expression (6) determines the shortest path to the efficient subset. For example, in Figure 2, the minimum contraction A requires to reach the efficient subset is given by the ratio AB/AH=1-BH/AH, which implies a reduction only in X_2. For unit D, the minimum contraction is given by the sum DE/DK+EF/EL, which implies a reduction in input X_1 to reach the isoquant plus a slack reduction in input X_2 to reach the efficient subset. Note that, for example, firm A can use two alternative reference points on the efficient subset, C and B, but B is closer to A than C. Therefore, firm A could prefer to visit B in order to improve its efficiency level.

<<<<<<<<<FIGURE 2 ABOUT HERE>>>>>>>>>>>

Unfortunately, the smallest contraction measure is not directly computable because, in general, there are many different paths to the efficient subset that must be compared. However, under the hypothesis of convexity of $L(\mathbf{u})$, it can be computed indirectly. For this purpose, we introduce the concept of *input-specific contraction*, in which a specific input determines the contraction path.

Definition 2. (*Input specific contraction*) The *k*-th input-specific contraction measures the contraction needed to reach the isoquant along the *k*-th axis. If the isoquant does not coincide with the efficient subset in the projection of the IOV, the measure considers the slacks in the rest of the inputs as additional contractions (to reach the efficient subset):

$$C(\mathbf{x},\mathbf{u})_k = max\left\{\sum_{n=1}^{N}(1-\theta_n):(x_1,\cdots,\theta_k x_k,\cdots,x_N)\in IsoqL(\mathbf{u})\wedge \right. \\ \left. \wedge(\theta_1 x_1,\cdots,\theta_N x_N)\in Eff\,L(\mathbf{u}) \quad ,\theta_n \le 1 \quad n=1,\cdots,N\right\} \tag{7}$$

Figure 2 illustrates this concept. The input-specific contraction in X_1 is DE/DK+EF/EL for unit D and AC/AJ for unit A. In the case of X_2, the contraction is DG/DI for unit D and AB/AH for unit A. Note that in the first case we add the slack EF/EL for unit D. Graphically it is clear, for the two dimensions case, that the smallest contraction defined in (6) must coincide with the smallest input-specific contraction. This is due to the assumption of convexity of $L(\mathbf{u})$. The distance to any point in the segment joining the two comparison points cannot be strictly smaller than either distance to these points. The same applies to the sum of contractions. For instance, the slope of the segment CB is less than one and, therefore, the closest point to A on the efficient subset is B. By convexity of $L(\mathbf{u})$ any point on the efficient subset between C and B must be at least as far from A as the points in the segment CB.

The analysis gets more complicated with more than 2 input dimensions (see Coelli, 1998). In this case there may be several efficient points that can be reached by slack contraction once the isoquant has been reached (for each input specific contraction). However, our N input specific contractions find the N reference points on the efficient subset that use the lowest possible quantity of each input (consuming no more quantity of the other inputs). By convexity, the part of the efficient subset that employs no larger quantities of any of the inputs than the unit evaluated must lie below the convex hull of these N reference points. An argument similar to the one outlined for the case of 2 dimensions shows that the smallest of those N measures is the shortest path to the efficient subset.

Proposition 1. If $L(\mathbf{u})$ is a convex set, the smallest contraction to the efficient subset is the smallest input-specific contraction[5]:

$$C(\mathbf{x},\mathbf{u}) = \min_n\left\{C(\mathbf{x},\mathbf{u})_n \quad , \quad n=1,\ldots,N\right\} \tag{8}$$

The measure proposed here differs from previous measures in significant ways. Its objective is not to find the largest feasible savings as the Farrell (1957), Färe (1975) or Färe and Lovell (1978) measures. Its objective is to find a relevant benchmark for efficiency improvement. For this purpose, Bogetoft and Hougaard (1999) have proposed to incorporate information about maximum feasible contractions in all inputs. Our proposed benchmark is markedly different. We try to find the closest reference on the production frontier. Thus, we are concerned with the smallest contractions, as are Frei and Harker (1999). Therefore, as Proposition 1 shows, the measure proposed in Definition 1 is similar to the smallest contraction measure proposed by Dervaux, Kerstens, and Vanden Eeckaut (1998), with the important difference that our benchmark lies on the efficient subset of the isoquant. It also differs from Frei and Harker (1999) in that relative contractions are used instead of absolute quantities of inputs.

5. Nonparametric Programming Model

To simplify the notation, we define S and N as the sets of output and input labels respectively, and J as the set of productive units in the sample. The subscript i will denote the IOV that is evaluated and λ the intensity vector, which represents the weights assigned to each unit j entered into the linear combination of units that define the efficient IOV.

The technique of Data Envelopment Analysis (DEA) introduced by Charnes, Cooper, and Rhodes (1978) and extended by Banker, Charnes, and Cooper (1984) has been widely used to measure the Farrell index of technical efficiency[7]. The k-th single-factor measure of efficiency can be computed by solving a transformed version of the original DEA program. We must take the i-th IOV back to the isoquant along the k-th input. The DEA program must find the linear combination of observed IOVs that minimizes the consumption of input k, constrained to use no more of the rest of the inputs and to produce no less outputs than the i-th IOV. Expression (10) shows the linear program that solves this problem:

$$
\begin{aligned}
\min_{\lambda,\theta} \quad & \theta_k \\
s.t. \quad & \sum_{j\in J} \lambda_j u_{js} \geq u_{is} && , \quad s \in S \\
& \sum_{j\in J} \lambda_j x_{jn} \leq x_{in} && , \quad n \in N, n \neq k \\
& \sum_{j\in J} \lambda_j x_{jk} \leq \theta_k x_{ik} \\
& \sum_{j\in J} \lambda_j = 1 \\
& \lambda_j \geq 0 && , \quad j \in J
\end{aligned}
\tag{10}
$$

However, there are two problems with the solution to this program. First, it may contain slacks in the inputs, which can be interpreted as additional feasible reductions. Second, even if we eliminate the slacks, some additional reductions may be feasible, because the *efficient* IOV used for comparison lies on the isoquant (not necessarily on the efficient subset). An alternative formulation that computes all feasible reductions is given by:

$$
\begin{aligned}
\min_{\lambda,\theta} \quad & M \cdot \theta_k + \sum_{\substack{n \in N \\ n \neq k}} \theta_n \\
s.t. \quad & \sum_{j \in J} \lambda_j u_{js} \geq u_{is} && , \quad s \in S \\
& \sum_{j \in J} \lambda_j x_{jn} \leq \theta_n x_{in} && , \quad n \in N \\
& \theta_n \leq 1 && , \quad n \in N, n \neq k \\
& \sum_{j \in J} \lambda_j = 1 \\
& \lambda_j \geq 0 && , \quad j \in J
\end{aligned}
\tag{11}
$$

where M is a large enough scalar to force the program to identify input k as the one defining the path to the isoquant. The search for feasible reductions in the rest of the inputs starts after the isoquant is reached, taking the i-th IOV down to the efficient subset. However, the weight assigned to input k also implies that values of $\theta_{-k} > 1$ are possible if they permit to obtain a lower θ_k. Therefore, the constraints $\theta_n \leq 1$ are necessary to ensure that reductions in input k are not achieved at the small cost (in terms of the objective function) of increasing some other input.

After computing θ in (11), the input-specific contraction defined in (7) can be derived as:

$$
C(\mathbf{x_i}, \mathbf{u_i})_k = \sum_{n \in N} (1 - \theta_n) \tag{12}
$$

In empirical applications it can be interesting to decompose the measure in (12) in two terms:

$$
C(\mathbf{x_i}, \mathbf{u_i})_k = (1 - \theta_k) + \sum_{\substack{n \in N \\ n \neq k}} (1 - \theta_n) \tag{13}
$$

where the first term represents the contraction to the isoquant and the second term the sum of the slacks.

6. Concluding remarks

In many contexts, technical inefficiency can be interpreted as the result of a lack of knowledge about certain critical aspects of the productive activity. In these cases, efficiency improvements may be achieved if the inefficient firm is able to learn better production routines. Benchmarking is a common tool used by firms that want to improve their understanding of the most successful practices in their field. However, the literature on technical efficiency has been more concerned with the problems of measuring inefficiency than with the problem of selecting relevant efficient benchmarks to learn from. The model proposed in this article can be used to identify a relevant benchmark for each inefficient firm in a sample. For each input k, we first compute the contraction to the isoquant along that input axis and then we add the remaining slacks that lead to the efficient subset. We refer to this sum as the *k*-th input-specific contraction. We show that the smallest input-specific contraction measure reveals the most similar set of efficient firms that can serve as a reference for the inefficient firm. This information can be incorporated into management programs in order to advise about which efficient firms should each inefficient firm visit in order to learn better managerial practices.

Notes

1. This radial notion of similarity has been used by Day, Lewin and Li (1995) to identify strategic groups in an industry. Using standard DEA, a firm is assigned to the strategic group defined by the firms in its comparison group (where inputs are strategies and output is a measure of performance).

2. For simplicity in the definitions, we will always assume that all the components of the IOVs are strictly positive.

3. We use the standard notation $\hat{\mathbf{x}} \leq \mathbf{x}$ to denote that $\hat{x}_n \leq x_n, n = 1 \cdots N \wedge \hat{\mathbf{x}} \neq \mathbf{x}$.

4. More elaborated transformations of these measures are discussed in Zieschang (1984) and Rusell (1985).

5. A formal proof is provided in the appendix.

6. See Färe, Grosskopf, and Lovell (1994) for a discussion of this technique.

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APPENDIX

In this appendix we prove that, under the assumption of convexity of $L(\mathbf{u})$, the smallest contraction to the efficient subset is the smallest input-specific contraction. From Figure 2, it is clear that the smallest distance from A to the efficient subset must be the distance from A to a reference point in the part of the efficient subset below the segment CB, where C and B are the reference points for the measurement of the two input-specific contractions. This is the reason for introducing the constraints $\theta_n \leq 1\ \forall n$ in the definition of (6). In the general case of N inputs, there are N reference points ($\mathbf{x}^1$,...,$\mathbf{x}^N$) one for each input-specific contraction and the smallest contraction must take a reference point in the part of the efficient subset below the convex hull of ($\mathbf{x}^1$,...,$\mathbf{x}^N$), to be using no larger quantities of any input. Note that this must be true if $L(\mathbf{u})$ is a convex set, because in this case the convex hull of ($\mathbf{x}^1$,...,$\mathbf{x}^N$), CB in Figure 2, belongs to $L(\mathbf{u})$ and therefore the efficient subset lies below (or on) it.

Thus, it suffices to prove that the contraction from the input vector ($\mathbf{x}^0$) to any point on the convex hull of ($\mathbf{x}^1$,...,$\mathbf{x}^N$) is bigger or equal to one of the input-specific contractions. The i[th] input-specific contraction would be:

$$C^i = \sum_{n=1}^{N}(1-\theta_n) = \sum_{n=1}^{N}\left(1-\frac{x_n^i}{x_n^0}\right) = N - \mathbf{x}^{\mathbf{i}}\bar{\mathbf{x}}^{0\,\prime}$$

where $\bar{\mathbf{x}}^{\mathbf{0}} = \left(\frac{1}{x_1^0}, \frac{1}{x_2^0}, \ldots, \frac{1}{x_N^0}\right)$

Take any arbitrary point $\tilde{\mathbf{x}}$ on the convex hull of ($\mathbf{x}^1$,...,$\mathbf{x}^N$):

$$\tilde{\mathbf{x}} = \sum_{i=1}^{N}\alpha_i \cdot \mathbf{x}^{\mathbf{i}} \qquad s.t. \sum_{i=1}^{N}\alpha_i = 1 \quad \wedge \quad \alpha_i \geq 0 \qquad \forall i$$

The contraction distance from $\mathbf{x}^{\mathbf{0}}$ to $\tilde{\mathbf{x}}$ is:

$$\tilde{C} = N - \tilde{\mathbf{x}}\bar{\mathbf{x}}^{\mathbf{0}\,\prime} = N - \sum_{i=1}^{N}\alpha_i\,\mathbf{x}^{\mathbf{j}}\,\bar{\mathbf{x}}^{\mathbf{0}\,\prime}$$

Claim: $\tilde{C} \geq C^i$ for some i=1...N.

Proof: The proof is by contradiction. Suppose $\tilde{C} < C^i \quad \forall i$. This implies:

$$N - \sum_{i=1}^{N}\alpha_i \cdot \mathbf{x}^i\bar{\mathbf{x}}^{\mathbf{0}\,\prime} < N - \mathbf{x}^i\bar{\mathbf{x}}^{\mathbf{0}\,\prime} \qquad \forall i$$

simplifying and multiplying both sides by α_i,and adding up over i, we get:

$$\sum_{i=1}^{N}\alpha_i\sum_{i=1}^{N}\alpha_i \cdot \mathbf{x}^i\bar{\mathbf{x}}^{\mathbf{0}\,\prime} > \sum_{i=1}^{N}\alpha_i\mathbf{x}^i\bar{\mathbf{x}}^{\mathbf{0}\,\prime}$$

and this implies $\sum\alpha_i > 1$, which is a contradiction, because as we know $\sum\alpha_i = 1$. This completes the proof■

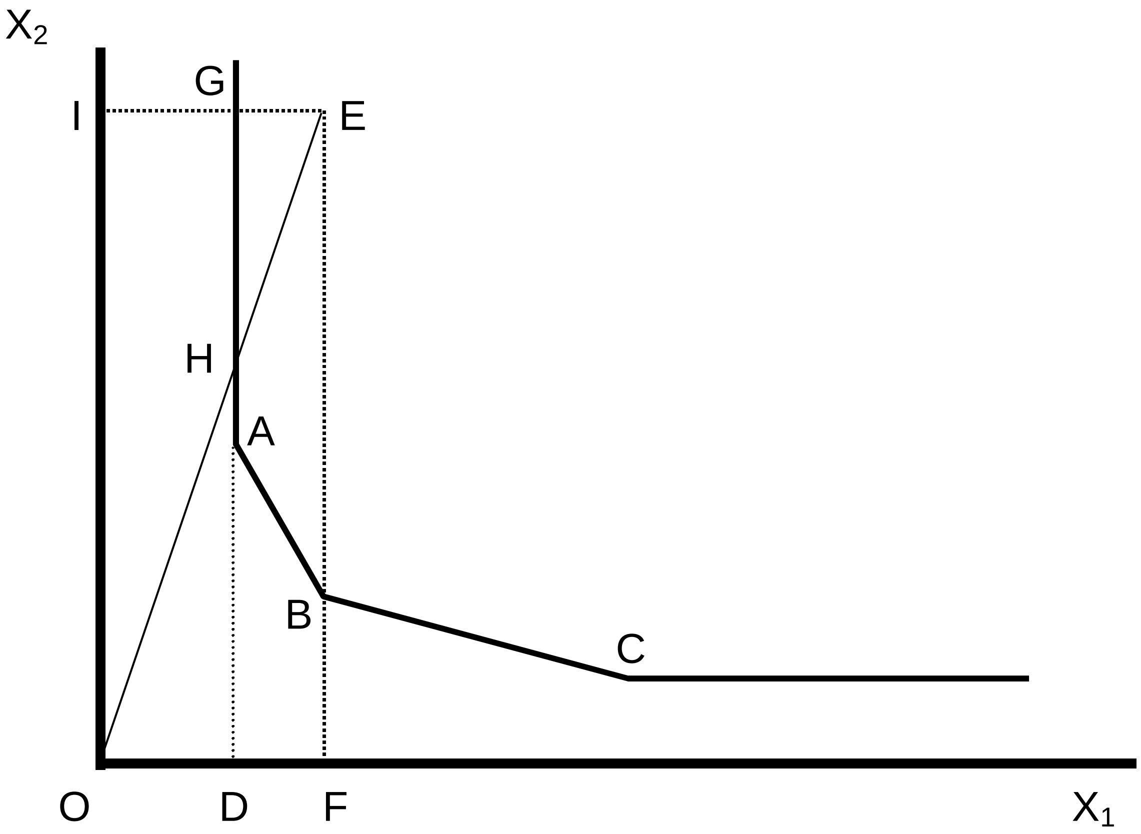

Figure 1. Current measures of technical efficiency

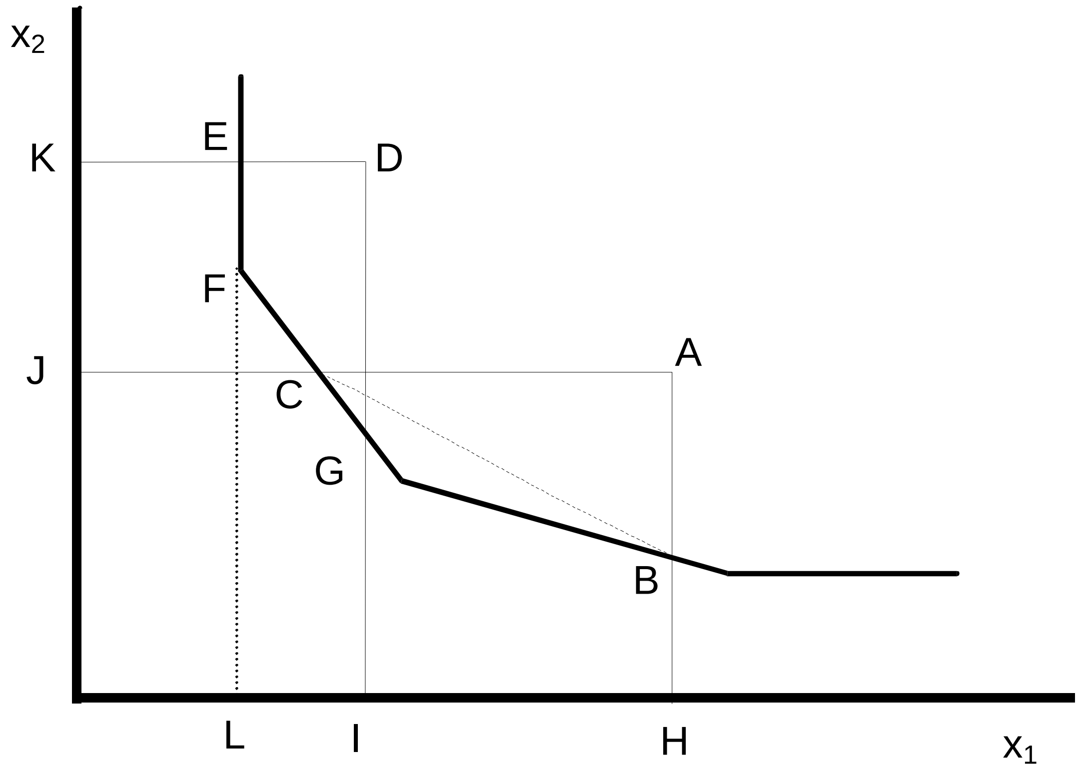

Figure 2. The smallest contraction to the efficient subset.